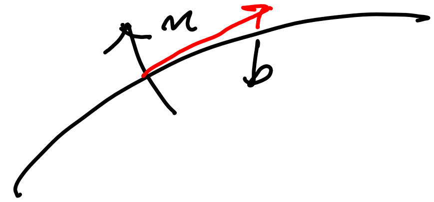


$$T = R^T \nabla_n \psi + \psi b$$

$$n = (\cos\theta, \sin\theta), \quad b = (\sin\theta, -\cos\theta)$$

$$\boxed{\begin{aligned} n_1 &= \cos\theta, \quad \frac{\partial n_1}{d\theta} = -\sin\theta = -n_2 \\ n_2 &= \sin\theta, \quad \frac{dn_2}{d\theta} = \cos\theta = +n_1 \end{aligned}}$$

$$\hat{\psi}(\theta) = \psi(n)$$

$$\psi(\lambda n) = \psi(n)$$

$$\frac{\partial \psi}{\partial n_1} n_1 + \frac{\partial \psi}{\partial n_2} n_2 = 0$$

$$-\frac{\partial \psi}{\partial n_1} n_2 + \frac{\partial \psi}{\partial n_2} n_1 = \frac{\partial \hat{\psi}}{\partial \theta}$$

rotation

$$\begin{pmatrix} n_1 & n_2 \\ -n_2 & n_1 \end{pmatrix} \nabla \psi = \begin{pmatrix} 0 \\ \frac{\partial \hat{\psi}}{\partial \theta} \end{pmatrix}$$

$$\nabla \psi = \begin{pmatrix} n_1 & -n_2 \\ n_2 & n_1 \end{pmatrix} \begin{pmatrix} 0 \\ \hat{\psi}_\theta \end{pmatrix}$$

$$R^T \nabla \psi = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} n_1 & -n_2 \\ n_2 & n_1 \end{pmatrix} \begin{pmatrix} 0 \\ \psi_\theta \end{pmatrix}$$

$$= \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \frac{\partial \hat{\psi}}{\partial \theta}$$

$$= \hat{\psi}_\theta n$$

K+L in

$$T = \psi_\theta n + \psi b$$ capillary vector

Equilibrium Equation

$$\frac{d}{ds} T = \frac{d}{ds}\left(\psi_\theta n + \psi b \right) \quad \text{on } \Gamma$$

$$\frac{db}{ds} = \kappa n \qquad \frac{dn}{ds} = -\kappa b$$

$$\frac{d}{ds} T = \frac{d}{ds}\left(\psi_\theta n + \psi b \right)$$

$$= \psi_{\theta\theta} \frac{d\theta}{ds} n + \psi_\theta \frac{dn}{ds} + \psi_\theta \frac{d\theta}{ds} b + \psi \frac{db}{ds}$$

$$= \psi_{\theta\theta} \kappa n + \cancel{\psi_\theta(-\kappa b)} + \cancel{\psi_\theta \kappa b} + \kappa n \psi$$

$$= \left(\psi_{\theta\theta} + \psi\right) \kappa n$$

Equilibrium:

$$\left(\psi_{\theta\theta} + \psi\right) \kappa = 0 \text{ on } \Gamma \qquad \kappa = \text{curvature}$$

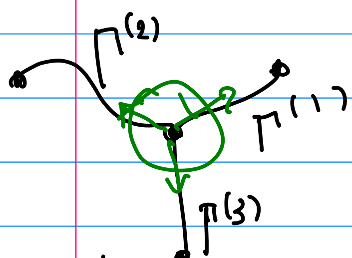
$$v_n n = \mu \frac{dT}{ds}$$ line segment

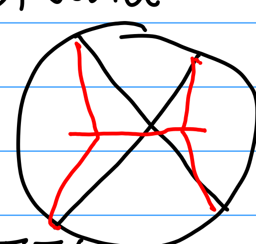
Evolution

$$v_n n = \mu \left(\psi_{\theta\theta} + \psi\right) \kappa n \text{ on } \Gamma$$

(+) parabolic

Equilibrium $\int_\Gamma \frac{d}{ds} T \, ds = T \Big|_{\partial\Gamma}$ "Steiner"





$\Rightarrow$ TJ's are stable

Triple junction

$$0 = \sum \int_{\Gamma^{(i)}} \frac{dT^{(i)}}{ds} ds$$

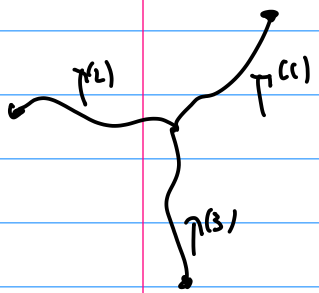
$$= \sum_{TJ} T^{(i)}$$

$$\boxed{\sum_{i=1}^{3} \left(\psi_\theta(\theta_i) n_i + \psi(\theta_i) b_i \right) = 0} \quad \text{Herring Condition}$$

Evolution

Herring is natural BC in equilibrium

Evolution: prescribe eq. BC



$$v_n = \mu(\psi_{\theta\theta} + \psi)\kappa \quad \text{on } \Gamma^{(i)}$$ Mullins

$$\sum \left(\psi_\theta n^{(i)} + \psi b^{(i)}\right) = 0 \quad \text{at TJ}$$

Mullins

Solve ? Bronsard + Reitich

K + Lin

single cell at equilibrium

Wulff Fonseca