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Hello

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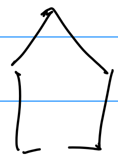

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Modeling "collisions".

Rarefied gas, large number of particles.

$f(t, x, \xi)$ - density of particles at time t at point x with speed ξ.

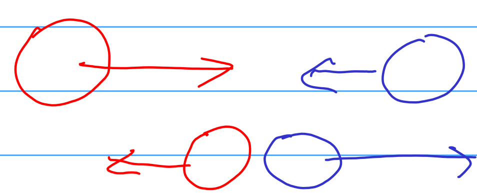
No collisions: $\partial_t f + \xi \cdot \nabla_x f = 0$

With collisions: $\partial_t f + \xi \cdot \nabla_x f = Q(f)$

collision term

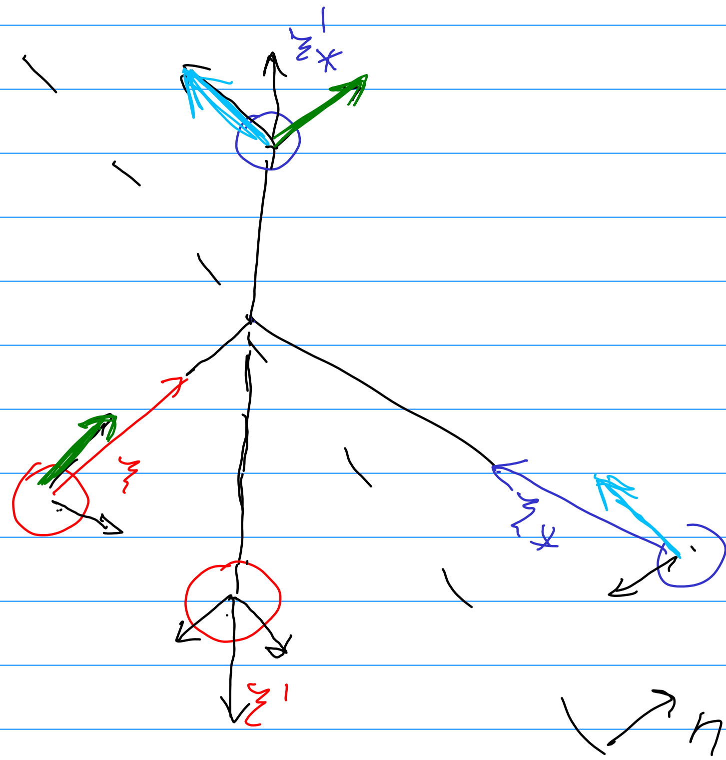
Elastic case: hard spheres same radius

(a)



velocities are exchanged

(b) in $\mathbb{R}^3$ – angle of collision is a unit vector n parallel to axis joining centers at collision



Components of velocities $\parallel n$ will be preserved, components $\perp n$ will be exchanged:

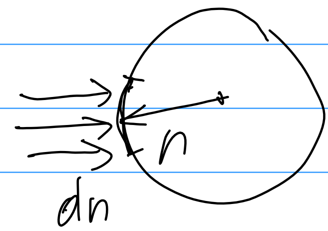
$$\begin{cases} \xi' = \xi - (n\cdot(\xi-\xi_*))n \\ \xi'_* = \xi_* + (n\cdot(\xi-\xi_*))n \end{cases}$$

new   old velocities

Exchange in $\perp n$ direction

$$\begin{cases} \xi'\cdot n = \xi_*\cdot n \\ \xi'_*\cdot n = \xi\cdot n \end{cases}$$

Deriving Boltzmann eqn:



Rate of collisions at an angle n is

$$|n \cdot V|\, dn$$

$$V = \xi - \xi_*$$

$\Rightarrow \ldots \Rightarrow$

Loss term: $$Q_-(f)(\xi) = \alpha \int_{\mathbb{R}^3} \int_{S^2} |n \cdot (\xi - \xi_*)| f(\xi) f(\xi_*)\, dn\, d\xi_*$$

Elastic collision is reversible so

$$|n(\xi' - \xi'_*)| f(\xi') f(\xi'_*) = |n(\xi - \xi_*)| f(\xi') f(\xi'_*)$$

$\Rightarrow$ Gain term:

$$Q_+ f(\xi) = \alpha \int_{\mathbb{R}^3} \int_{S^2} |n \cdot (\xi - \xi_*)| f(\xi') f(\xi'_*)\, dn\, d\xi_*$$

Boltzmann eqn:

$$\partial_t f + \xi \cdot \nabla_x f = \int_{\mathbb{R}^3} \int_{S^2} (f' f'_* - f f_*) |n \cdot (\xi - \xi_*)|\, dn\, d\xi_*$$

H-theorem:

Rewrite Boltzmann as

$$\partial_t \mathcal{H}(t,x) + \nabla_x \cdot \mathcal{I}(t,x) = W(t,x)$$

where $\mathcal{H}(t,x) = \int_{\mathbb{R}^3} f \ln f \, d\xi = -S$

Shannon-Boltzmann entropy

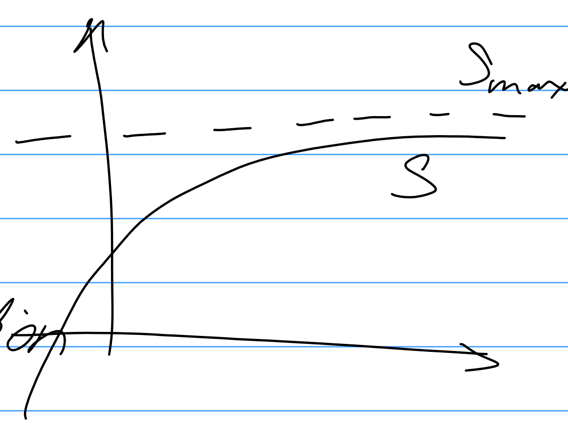
$$\mathcal{I} = \int_{\mathbb{R}^3} \xi f \ln f \, d\xi$$

$$W = \int_{\mathbb{R}^3} Q(f) \ln f \, d\xi$$

$\Rightarrow$ Since $W \leq 0$ and f decays at $|x| \to \infty$

$$\Rightarrow \boxed{\frac{d}{dt}\mathcal{H}(t) \leq 0}$$

Max entropy solution is a Gaussian distribution



Beyond elastic case:

Granular flow

$v_1 - v_2 = -r(u_1 - u_2)$

new vel. old vel.

$r = \begin{cases} 0 & \text{; inelastic case} \\ 1 & \text{; elastic case} \end{cases}$

$(u_1, u_2) \to (pu_1 + qu_2,\ qu_1 + pu_2)$;

$p + q = 1, \quad r = 1 - 2p$

$K(v_1, v_2) = |v_1 - v_2|^\gamma$

collision rate

$\gamma = 0 \Rightarrow f(v) \to \delta(v)$

$$T = T_0 e^{-\lambda_2 t}$$

exp. decay of temperature

$\lambda_n = 1 - p^n - q^n$

Smoluchowski coagulation

$V_1 + V_2 \to V^*$

mass $m^* = m_1 + m_2$

$p^* = p_1 + p_2$

momentum

Kinetic energy is dissipated

Elastic	Granular	Smoluchowski
Mass = const	Mass = const	Mean mass grows
$E_{kinetic}$ = const	$E_{kin} \searrow$	$E_{kin} \searrow$ (cooling effect)
Reversible	Irreversible collision	