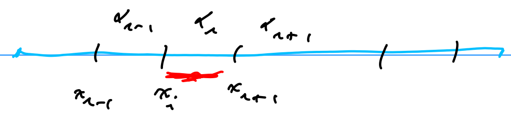
Simplified Problem



$\psi(\alpha)$

$$E(t) = \sum \psi(\alpha_i)(x_{i+1} - x_i) = \sum \psi(\alpha_i)\, l_i(\alpha, t)$$

- enhancing role of triple junctions
- suppressing growth by coarsening

$$\frac{dx_i}{dt} = -\frac{\partial E}{\partial x_i} : \qquad \frac{dx_i}{dt} = \psi(\alpha_{i-1}) - \psi(\alpha_i)$$

a $$\frac{dE}{dt} = -\sum\left(\frac{dx_i}{dt}\right)^2 = -\sum\left(\psi(\alpha_{i+1}) - \psi(\alpha_i)\right)^2 \leq 0$$

in the absence of rearrangement events

check: E continuous, decreasing, Lipschitz

b $$\int_0^\tau \sum\left(\frac{dx_i}{dt}\right)^2 dt + E(\tau) \leq E(0)$$

$\rho(\alpha, t)$ GBCD

$$\gamma_0 \int_\Omega \int_0^\tau \left(\frac{\partial \rho}{\partial t}\right)^2 dt\, d\alpha + \int_\Omega \rho \psi \, d\alpha \Big|_{t=\tau} \leq \int_\Omega \rho\psi\, d\alpha \Big|_{t=0}$$

$$\gamma_0 \int_0^\tau \int_\Omega \left(\frac{\partial \rho}{\partial t}\right)^2 d\alpha\, dt + F_\lambda(\rho)\Big|_{t=\tau} \leq F_\lambda(\rho)\Big|_{t=0}$$

$$F_\lambda(\rho) = \int_\Omega \rho\psi\, d\alpha + \lambda \int_\Omega \rho \log \rho\, d\alpha$$

not proper dissipation

VCZ: ensemble of spring-mass-dashpots $\qquad \int_0^\tau \int_\Omega v^2 f\, d\alpha\, dt$

Try to estimate

look at $\rho(x,t)$ $\qquad \rho(x,0) = \rho^*$ $\qquad \rho(x,\tau) = \rho$

induces a transfer function $\phi(x,t)$

$$\int_\Omega \rho(x,t)\, \zeta(x)\, dx = \int_\Omega \zeta(\phi(x,t))\, \rho^*(x)\, dx$$

$v(x,t)$ $\qquad \phi_t(x,t)$

$$\rho_t + (v\rho)_x = 0$$

$$F(x,t) = \int_{-\infty}^{x} \rho(x',t)\,dx' \quad \text{distribution function}$$

$$\frac{\partial F}{\partial t} + v\frac{\partial F}{\partial x} = \frac{\partial F}{\partial t} + v\rho = 0 \quad (\text{constant of integration} = 0)$$

$$v^2 \rho \approx \frac{F_t^2}{\rho} \leq \frac{x}{\rho} \cdot \int_\Omega \rho_t^2(y,t)\,dy$$

appears in the form

$$\int_0^\tau \int_\Omega v^2 \rho \,dx\,dt \leq \frac{C_\Omega}{\min \rho} \cdot \int_0^\tau \int_\Omega \rho_t^2 \,dy\,dt$$

Benamou – Brenier gives Wasserstein metric

$$\frac{\mu}{2} \int_0^\tau \int_\Omega v^2 \rho \,dx\,dt + F_\lambda(\rho)\Big|_{t=\tau} \leq F_\lambda(\rho)\Big|_{t=0}$$

Assume realized by infimum

$$\frac{\mu}{2\tau} d(\rho,\rho^*)^2 = \inf \int_0^\tau \int_\Omega v^2 \rho \,dx\,dt$$

take infimum again $\Longrightarrow$

$$\frac{\mu}{2\tau} d(\rho,\rho^*)^2 + F_\lambda(\rho) = \inf_\eta \left\{ \frac{\mu}{2\tau} d(\eta,\rho^*)^2 + F_\lambda(\eta) \right\}$$

For each τ determine $\{\rho^{(k)}\}$ iteratively minimize

$$\rho^{(\tau)}(x,t) = \rho^{(k)}(x) \quad \text{in } \Omega, \quad k\tau \leq t < (k+1)\tau$$

let $\tau \to 0$

limit ρ solves

$$\mu \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial \rho}{\partial x} + f\rho\right)$$

Quest: find λ

$\rho(\alpha,t) \longrightarrow \rho_\lambda(\alpha)$ as $t \to \infty$ for correct λ

convergence is exponentially fast

Kullback-Leibler relative entropy

$$\rho_\lambda(\alpha) = \frac{1}{Z_\lambda} e^{-\frac{\psi(\alpha)}{\lambda}} , \quad Z_\lambda = \int_\Omega e^{-\frac{\psi(\alpha)}{\lambda}} d\alpha$$

$$\Phi_\lambda(\eta) = \lambda \int_\Omega \eta \log \frac{\eta}{\rho_\lambda} d\alpha$$

$$\eta \geqq 0, \quad \int_\Omega \eta \, d\alpha = 1$$

$$\Phi_\lambda(\eta) = F_\lambda(\eta) + \lambda \log Z_\lambda$$

choosing $\lambda = \sigma$ for which $\Phi_\sigma(\rho) \longrightarrow 0$ OR

$$\Phi_\sigma(\rho) \longrightarrow \min_\lambda \Phi_\lambda(\rho) \text{ at } \text{"}t = \infty\text{"}$$

$$\Phi_\lambda(\eta) = \lambda \int_\Omega \eta \log \eta \, d\alpha - \lambda \int_\Omega \eta \log \rho_\lambda \, d\alpha$$

$$= \lambda \left\{ \int_\Omega \left(-\frac{1}{\lambda} \log \rho_\lambda\right) \eta \, d\alpha + \int_\Omega \eta \log \eta \, d\alpha \right\}$$

$$\psi_\lambda = \psi + \frac{1}{\lambda} \log Z_\lambda$$

$$= \lambda \left\{ \int_\Omega \psi_\lambda \eta \, d\alpha + \int_\Omega \eta \log \eta \, d\alpha \right\} \qquad \int_\Omega e^{-\psi_\lambda} d\alpha = 1$$

looks like a prefix code

determine σ by asking for optimal prefix code